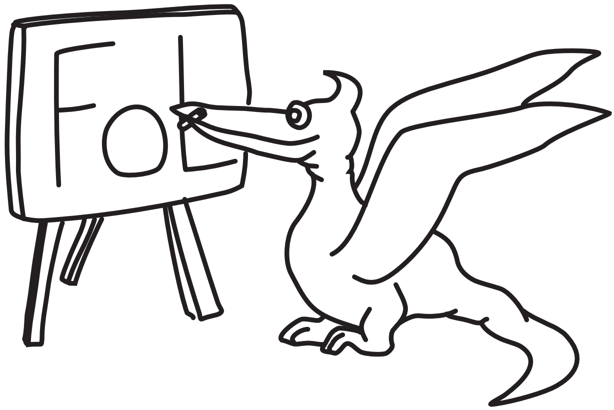
Solution of 2^{nd} Online Physics Brawl



Problem FoL.1 ... curtain

Imagine a thin rigid homogeneous rod with mass $m = 1\,\mathrm{kg}$ and length $l = 2\,\mathrm{m}$, which is attached to a horizontal rail by a small massless ring at its end, so that it can slide without friction. As the ring accelerates at a constant rate $a = 5\,\mathrm{m{\cdot}s^{-2}}$, the rod makes a constant angle φ to the vertical. Find this angle given that the whole situation takes place on the Earth's surface, thus in the presence of the vertical acceleration due to gravity $g = 10\,\mathrm{m{\cdot}s^{-2}}$, neglecting the effects of air resistance. State your answer in radians. *Náry, looking at the mechanics of curtain.*

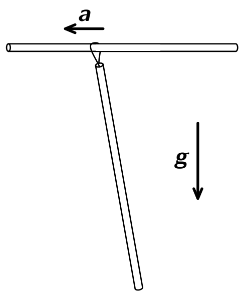


Fig. 1: To the problem 1

Let us describe the system in the frame of reference comoving with the ring. In this frame, the rod has only one degree of freedom, namely the angle φ, which was said to be constant. Hence the rod can be considered at rest with respect to this frame and we are left with determining the conditions for equilibrium. Apart from the force due to gravity, there is a ficticious force acting on every element of the rod, which is due to the accelerating frame of reference. Resolving the forces acting on the rod in the direction perpendicular to it (the parallel components are compensated for by the tension in the rod) and equating the corresponding moments, we find the following condition for the rod to be at equilibrium

$$\operatorname{tg}\varphi = \frac{a}{g}\,.$$

Substituting the numerical values for a and g, we obtain $\varphi \doteq 0.46\,\mathrm{rad}$.

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Problem FoL.2 ... hangmen

There are two balls of mass m and electric charge q (with the same sign) hanging from two slings of length l fixed in the same point. These balls are placed in air with density $\varrho_\mathrm{a} = 1.2\,\mathrm{kg{\cdot}m^{-3}}$. Due to the electric repulsion of the balls, the slings are forming an angle α. If we put the same aparate in the olive oil with relative permitivity $\varepsilon_\mathrm{r} = 3$ and density $\varrho_o = 900\,\mathrm{kg{\cdot}m^{3}}$, the angle will remain the same. Consider the permitivity of the air to be same as permitivity od vacuum. What is the density of the balls?
f(Aleš) was reading about little balls and decided to recompute the problem for nicer situation.

There are two forces affecting the balls, one electric and the other one gravitational. Electric force can be computed from Coulomb's law and the gravitational which can be computed from law of gravity, hence

$$F_\mathrm{e} = \frac{1}{4\pi\varepsilon_0\varepsilon_\mathrm{r}} \frac{q^2}{4\sin^2\left(\frac{\alpha}{2}\right) l^2}\,,$$

$$F_g = V\left(\varrho_\mathrm{b} - \varrho_\mathrm{e}\right) g\,,$$

where ϱ_e is the density of the environment.

Those two forces are perpendicular to each other. Gravitational force is vertical while the electric force is horizontal.

From geometry

$$\frac{F_\mathrm{e}}{F_g} = \mathrm{tg}\left(\frac{\alpha}{2}\right).$$

Let's mark the forces after inserting the aparate to the olive oil with primes. Condition for the constant angle gives us

$$\frac{F_\mathrm{e}}{F_g} = \frac{F'_\mathrm{e}}{F'_g}\,,$$

from here

$$\frac{1}{\varrho_\mathrm{b} - \varrho_\mathrm{a}} = \frac{1}{\left(\varrho_\mathrm{b} - \varrho_o\right)\varepsilon_\mathrm{r}}\,,$$

where we assumed that the permitivity of the air is the same as of vacuum.

Then

$$\varrho_\mathrm{b} = \frac{\varrho_\mathrm{o}\varepsilon_\mathrm{r} - \varrho_\mathrm{a}}{\varepsilon_\mathrm{r} - 1}\,.$$

Plugging in numbers, $\varrho_\mathrm{b} \doteq 1349,4\,\mathrm{kg\cdot m^{-3}}$.

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Problem FoL.3 ... blue rock

Center of mass of a mountaineer climbing on a rock is $h = 24\,\mathrm{m}$ *above ground. The last belay (place where the climber's rope runs through a metallic circle attached to the rock) is at a height* $h_0 = 20\,\mathrm{m}$. *The climber slips and falls. How closest to the ground does he get during his fall? Young's modulus of the rope is* $E = 100\,\mathrm{MPa}$, *its radius* $r = 0.5\,\mathrm{cm}$ *and mass of the climber* $m = 70\,\mathrm{kg}$. *Neglect mass of the rope and all friction. Assume that the rope is attached to the climber in his center of mass. All distances are given with respect to a securing device which is attached to the ground and does not move during the fall. The local gravitational acceleration is* $g = 9.81\,\mathrm{m\cdot s^{-2}}$. *Honza thought carefully before climbing.*

Let's denote l the increase in the length of the rope during the fall. From the conservation of energy we have

$$mg\left(2\left(h - h_0\right) + l\right) = \frac{1}{2}\frac{E\pi r^2}{h} l^2\,.$$

Solving the quadratic equation we find l. The height h_f in which the climber stops falling is then $h_\mathrm{f} = h - 2(h - h_0) - l \doteq 7.7\,\mathrm{m}$.

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Problem FoL.4 ... gas block

There is a movable divider in a closed cylindrical vessel, separating it into two chambers. One of the chambers contains $25\,\mathrm{mg}$ *of* N_2 *while the second one contains* $40\,\mathrm{mg}$ *of* He. *Assume that the equilibrium state was attained. What is the ratio of the lenghts of the chambers in the equilibrium state? Assume ideal-gas behaviour. Your answer should be less than 1.*

Janapka was going through her old exercise books.

The pressure in both chambers is the same, since we are in equilibrium state. Let us compute the amount of substance in both chambers. We need to look up molar masses M in the tables. For nitrogen, it is $28\,\mathrm{g\cdot mol^{-1}}$ while for helium, we have $4\,\mathrm{g\cdot mol^{-1}}$. We will plug in the numbers into the formula for the amount of substance, where $n = m/M$ and m is the mass of the gas in the chamber. Let us denote the area of the base of the bottle by S and the lenghts by L. Then, taking into account the ideal gas law, we can write

$$p = \frac{n_1 RT}{SL_1} = \frac{n_2 RT}{SL_2}\,,$$

whence $L_1/L_2 = n_1/n_2 \doteq 0.089$.

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Problem FoL.5 ... marble play

We want to galvanise (in copper sulphate solution) an iron sphere of mass $m_{\mathrm{Fe}} = 8\,\mathrm{kg}$ *and density* $\varrho_{\mathrm{Fe}} = 8\,\mathrm{g\cdot cm^{-3}}$. *We aim for a layer of copper* $\Delta r = 2\,\mathrm{mm}$ *thick. The density of copper is* $\varrho_{\mathrm{Cu}} = 9\,\mathrm{g\cdot cm^{-3}}$, *molar mass is* $M_{\mathrm{Cu}} = 63.5\,\mathrm{kg\cdot mol^{-1}}$ *and we apply constant current* $I = 0.5\,\mathrm{A}$. *How long does the process take? State your answer in days, ceiled (i.e. 3.238 days are 4 days).*

Kiki was remembering the chemistry olympiad.

Essential for the solution is finding out how much copper m_{Cu} is deposited onto the surface of the sphere during the electrolysis. We know the density of the sphere and its mass, hence we can compute its volume, which is $V_{\mathrm{Fe}} = m_{\mathrm{Fe}}/\varrho_{\mathrm{Fe}}$. The volume can also be expressed as $V = 4\pi r^3/3$, so we can infer the radius of the sphere r, which will grow to $R = r + \Delta r$, where $\Delta r = 0.002\,\mathrm{m}$. The volume of the resulting sphere will be $V = 4\pi R^3/3$. Copper and iron do not mix in the process, so we can deduce the volume of the copper layer, which is $V_{\mathrm{Cu}} = V - V_{\mathrm{Fe}}$. The mass of the layer can be computed as $m_{\mathrm{Cu}} = V_{\mathrm{Cu}}\varrho_{\mathrm{Cu}}$. We can input the known quantities to obtain

$$\begin{aligned}
m_{\mathrm{Cu}} &= V_{\mathrm{Cu}}\varrho_{\mathrm{Cu}}\,,\\
m_{\mathrm{Cu}} &= \left(V - V_{\mathrm{Fe}}\right)\varrho_{\mathrm{Cu}}\,,\\
m_{\mathrm{Cu}} &= \left(\frac{4}{3}\pi R^3 - \frac{m_{\mathrm{Fe}}}{\varrho_{\mathrm{Fe}}}\right)\varrho_{\mathrm{Cu}}\,,\\
m_{\mathrm{Cu}} &= \left(\frac{4}{3}\pi\left(\sqrt[3]{\frac{3m_{\mathrm{Fe}}}{4\pi\varrho_{\mathrm{Fe}}}} + \Delta r\right)^3 - \frac{m_{\mathrm{Fe}}}{\varrho_{\mathrm{Fe}}}\right)\varrho_{\mathrm{Cu}}\,.
\end{aligned}$$

We will use Faraday's laws of electrolysis to infer the time needed to produce such amount of copper. We write $m = AIt$, where A is the corresponding electrochemical equivalent $A = M_{\mathrm{Cu}}/(Fz)$, $M_{\mathrm{Cu}} = 0.0635\,\mathrm{kg\cdot mol^{-1}}$ is the molar mass of copper, $F = 96\,485\,\mathrm{C\cdot mol^{-1}}$ is the

Faraday's constant and $z = 2$ is the number of electrons released during the reaction when a copper cation with oxidation number II changes into a copper atom with oxidation number 0. The time elapsed can be computed as

$$t = \frac{m_\mathrm{C} u}{AI}\,,$$
$$t = \frac{m_\mathrm{C} u F z}{I M_\mathrm{C} u}\,.$$

Plugging in the numbers, we get $t \doteq 5\,462\,\mathrm{ks}$, which is equivalent to 62 days after ceiling.

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Problem FoL.6 ... springy one

We happen to have forgotten a spring with natural length $l = 0.5\,\mathrm{m}$ and a spring constant of $k = 5\,\mathrm{N\cdot m^{-1}}$ in the outer space. There is a mass $m_1 = 1\,\mathrm{kg}$ attached to one end of the spring while at the other end, there is a mass $m_2 = 3\,\mathrm{kg}$. Compute the period of small oscillations of the spring. *Paťo was going through his old exercise book.*

The centre of mass of an isolated system considered with respect to an inertial frame of reference cannot accelerate. Hence, in the centre of mass reference frame, the periods of oscillations of both masses must be the same. The centre of mass is at the distance

$$x = l\frac{m_2}{m_1 + m_2}$$

from m_1. Let us regard the spring as consisting of two springs connected in series at the centre of mass. The spring constants of these springs are inversely proportional to their respective lengths (the shorter spring is more rigid than the longer one), hence for the spring connected to m_1 we have

$$k_1 = k\frac{l}{x} = k\frac{m_1 + m_2}{m_2}\,.$$

The period of oscillations of the whole system is equal to the period of oscillations of this part of it, so

$$T = 2\pi\sqrt{\frac{m_1}{k_1}} = 2\pi\sqrt{\frac{m_1 m_2}{k(m_1 + m_2)}}\,.$$

Plugging in the numbers, we get $T \doteq 2.43\,\mathrm{s}$.

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Problem FoL.7 ... expandable balloon

A closed helium balloon is taking off from the Earth's surface, where the temperature and pressure are $300\,\mathrm{K}$ and $101\,\mathrm{kPa}$ respectively. Eventually, it will reach the point where the temperature and pressure are $258\,\mathrm{K}$ and $78\,\mathrm{kPa}$ respectively. Assuming that the balloon is of spherical shape initially with radius $10\,\mathrm{m}$, find the factor by which its radius will increase. The balloon is in thermodynamic equilibrium with its environment. Do not take into account the surface tension of the balloon. *Tomáš was dreaming about flying in a hot air balloon.*

We can use the ideal gas law to write

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}\,.$$

Hence the ratio V_2/V_1 can be expressed as

$$\frac{p_1 T_2}{p_2 T_1} \doteq 1.114\,.$$

The initial volume of the balloon is $V_1 = 4\pi r_1^3/3$. We will express the new volume as

$$V_2 = V_1 \frac{p_1 T_2}{p_2 T_1} = \frac{4\pi}{3} r_2^3 = \frac{4\pi}{3} r_1^3 \cdot \frac{p_1 T_2}{p_2 T_1}\,.$$

Hence the resulting ratio is

$$\sqrt[3]{\frac{p_1 T_2}{p_2 T_1}} \doteq 1.037\,.$$

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Problem FoL.8 ... resistant prism

The edges of a square pyramid are made out of wires which are conductively connected at all vertices. Compute the resistance across the opposite vertices on a diagonal of the base square, given that the resistance of one metre of the wire is $1\,\Omega$*, the height of the pyramid is* $\sqrt{7}\,\mathrm{m}$ *and the base length is* $2\,\mathrm{m}$*.* *Pikoš, while marking solutions.*

We want to find the resistance across the opposite vertices of the base of a square pyramid. By symmetry of the problem, we note that all the remaining vertices are at the same potential, including the top of the pyramid. Hence there is no current through the wires mutually connecting these vertices and we can discard them. We are left with three pairs of resistors connected in parallel, each pair containing identical resistors. As for the two pairs defining the base, the resistors have resistance $2\,\Omega$ while in the case of the third pair, the resistance of the constituent resistors is (by Pythagoras' theorem)

$$\sqrt{\left(\sqrt{7}\right)^2 + \left(\frac{\sqrt{2^2+2^2}}{2}\right)^2}\,\Omega = 3\,\Omega\,.$$

Thus the sought-after resistance is $1.5\,\Omega$.

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Problem FoL.9 ... satellite

Consider a planet and its natural satellite orbiting about their common centre of mass, where the motion takes place in a plane. The magnitude of the tangent velocity of the satellite, defined with respect to the centre of mass of the system, is $2.5\,\mathrm{km\cdot s^{-1}}$. *Find the ratio of the mass of the planet to the mass of the satellite in order for the centre of mass of the system to be located at the planet's surface, given that the orbits are circular. The mass of the planet is* $M_\mathrm{p} = 7.6\cdot 10^{24}\,\mathrm{kg}$, *it has a radius of* $R_\mathrm{p} = 7\,436\,\mathrm{km}$ *and the radius of the satellite is* $R_\mathrm{m} = 1\,943\,\mathrm{km}$

Nicola was thinking about the two-body problem.

Since we know the speed of the moon in its orbit, we can derive the distance h of the moon's center from the planet's surface. We know that $F_G = F_\mathrm{d}$, where F_G is the magnitude of the force due to gravity on the moon and F_d is the magnitude of the centripetal force making the moon follow the circular orbit. Hence we have

$$\kappa \frac{M_\mathrm{p} M_\mathrm{m}}{(R_\mathrm{p}+h)^2} = \frac{M_\mathrm{m} v^2}{h}\,,$$

where κ is the gravitational constant. Solving the above equation for h, we have

$$0 = h^2 + \left(2R_\mathrm{p} - \frac{\kappa M_\mathrm{p}}{v^2}\right) h + R_\mathrm{p}^2\,. \tag{1}$$

Denoting $x = h/R_\mathrm{p}$, we obtain

$$0 = x^2 + \left(2 - \frac{\kappa M_\mathrm{p}}{v^2 R_\mathrm{p}}\right) x + 1\,.$$

We need the centre of mass at the surface of the planet. By definition of the centre of mass, we write

$$R_\mathrm{p} = \frac{(h+R_\mathrm{p})M_\mathrm{m}}{M_\mathrm{m}+M_\mathrm{p}}\,.$$

Thus, multiplying through by $M_\mathrm{m} + M_\mathrm{p}$ and dividing by $M_\mathrm{m} R_\mathrm{p}$, we get

$$\frac{M_\mathrm{p}}{M_\mathrm{m}} + 1 = x + 1 \quad \Rightarrow \quad x = \frac{M_\mathrm{p}}{M_\mathrm{m}}\,.$$

By solving the equation (1), we get $x = M_\mathrm{p}/M_\mathrm{m} \doteq 8.79$.

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Problem FoL.10 ... a loop

Consider an electron-emitting source with an emitting speed of $v = 1.5\cdot 10^7\,\mathrm{m\cdot s^{-1}}$. *At a point* P, *the electrons enter a homogeneous magnetic field with a magnitude of* $B = 1\cdot 10^{-3}\,\mathrm{T}$. *The vector of electrons' velocity at* P *makes an angle* $\varphi = 15^\circ$ *to the magnetic field vector. Find the distance of* P *from the point where the electrons again (for the first time) cross the field line going through* P.

Zdeněk has teleported inside a monitor.

We need to resolve the velocity vector v into two components. One component is perpendicular to the field lines, $v_x = v \sin \varphi$, while the other one is parallel to them, $v_y = v \cos \varphi$. The Lorentz force is acting as a centripetal force, so

$$Qv_x B = \frac{mv_x^2}{r} \,,$$

whence we can express r as

$$r = \frac{mv_x}{QB} \,.$$

Then the period of the orbital motion can be obtained as

$$T = \frac{2\pi r}{v_x} = \frac{2\pi m}{QB} \,.$$

This is basically the time the electron needs to cross the field line once more. In the meantime, it will travel through a distance s in the direction parallel to the field lines

$$s = v_y \frac{2\pi m}{QB} \,.$$

Substituting for the charge and the mass of the electron, we get $s \doteq 0.52\,\mathrm{m}$.

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Problem FoL.11 ... ecologically passive

A passionate tree hugger (weighing only $m = 50\,\mathrm{kg}$) learned that the city council decided to chop down his favourite tree. He climbed onto the top of his homogeneous green friend believing that he would keep the tree killers away. However, the lumberjacks came and cut down the $h = 10\,\mathrm{m}$ tall tree weighing $M = 1\,\mathrm{t}$. What was the speed with which the tree hugger, initially resting at the tree top, hit the ground? The acceleration due to gravity is $g = 9.81\,\mathrm{m{\cdot}s^{-2}}$.

Terka, while climbing a tree.

The crucial one is the law of conservation of energy here. The potential energy of the tree hugger and the tree is

$$E_\mathrm{p} = g\left(mh + \frac{h}{2}M\right) \,.$$

The kinetic energy is given by the moment of inertia I of the system as

$$I = \frac{1}{2}I\omega^2 \,,$$

where ω is the angular speed of the tree hitting the ground. The moment of inertia can be computed with aid of the parallel axis theorem as

$$E_\mathrm{k} = \frac{1}{12}Mh^2 + M\left(\frac{h}{2}\right)^2 + mh^2 \,.$$

Conserving the total energy, we have

$$hg\,(2m + M) = \left(\frac{1}{3}M + m\right)h^2\omega^2 \,.$$

Whence we obtain ω as well as $v = \omega h$:

$$v = \sqrt{\frac{3M + 6m}{M + 3m} gh}\,.$$

Plugging in the numbers, we get $v \doteq 16.8\,\mathrm{m\cdot s^{-1}}$.

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Problem FoL.12 ... nucleus on a diet

Imagine we have a nucleus with nucleon number $A = 36$, proton number $Z = 17$ and the mass of $m_\mathrm{k} = 5.99965 \cdot 10^{-26}\,\mathrm{kg}$ What would be the sum of bonding energies in nucleus in one mole of nuclei of this element? Provide your answer in TJ.

Kiki was bored during the lecture on anorganic chemistry.

The element contains $Z = 17$ protons and $N = A - Z = 19$ neutrons. Mass of proton as a standalone particle is $m_\mathrm{p} = 1.6725 \cdot 10^{-27}\,\mathrm{kg}$, mass of neutron is $m_\mathrm{n} = 1.6749 \cdot 10^{-27}\,\mathrm{kg}$. Just adding these two, the nucleus should weigth $m_\mathrm{t} = 17m_\mathrm{p} + 19m_\mathrm{n}$. The real mass of the nucleus is smaller. The difference is directly proportional to bonding energy

$$\delta m = m_\mathrm{t} - m_\mathrm{k}$$

according to $E = \delta m\, c^2$. We are interested in energy of one mole of the nuclei, $E_\mathrm{m} = N_A \cdot E$, where $N_A = 6.022 \cdot 10^{23}\,\mathrm{mol^{-1}}$ is Avogadro number. Plugging in the numbers, we get $E_\mathrm{m} \doteq 1.40 \cdot 10^{13}\,\mathrm{J} = 14.0\,\mathrm{TJ}$.

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Problem FoL.13 ... music of the locomotives

Two locomotives A and B are moving with velocities $v_A = 15\,\mathrm{m\cdot s^{-1}}$ in the right direction and $v_B = 30\,\mathrm{m\cdot s^{-1}}$ in the left direction, facing each other on the paralel railways. Locomotive A whistles on a frequence 200 Hz. The speed of the sound is $340\,\mathrm{m\cdot s^{-1}}$. Let's assume that some of the sonic waves will be reflected from the locomotive B back to locomotive A. Which frequency will be heard by the engineer in the locomotive A? Assume that rightwards direction is positive and the environment is not moving. *Janapka was playing with trains.*

The problem makes use of Doppler effect. First, let's compute the frequency which will be heard by engineer in locomotive B. Locomotive A is moving to the right, in the positive direction, so locomotive B will have velocity $-|v_B|$. Environment is not moving. The velocity of the source is equal to the velocity of locomotive A, moving in the positive direction. The speed of sound is denoted as v. Frequency which is heard by engineer in locomotive B is

$$f_B = f_0 \frac{c + |v_B|}{c - |v_A|} \doteq 228\,\mathrm{Hz}\,.$$

This is the frequency sent from locomotive B to locomotive A. So let's apply Doppler's law once more to get the final frequency

$$f_A = f_B \frac{c + |v_A|}{c - |v_B|} \doteq 261\,\mathrm{Hz}\,.$$

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Problem FoL.14 ... capacitors in the circuit

Voltage in the circuit on the attached image is 10 V *and each capacitor has capacity* 10 µF. *Compute the charge (in* µC*) on the capacitor* C_1. *Dominika on a branch.*

We use the numbering of capacitors as in the image. Let's recall the rules for computing the final capacity if the capacitors are in series (C_s) or parallel (C_p)

$$\frac{1}{C_\mathrm{s}} = \frac{1}{C_a} + \frac{1}{C_b}\,, \quad C_\mathrm{p} = C_a + C_b .$$

Using this formula, we replace the capacitors $1, 2, 3$ with equivalent C_{123} and capacitors $1, 2, 3, 4$ with equivalent C_{1234}

$$C_{123} = \frac{3}{2}C, \qquad C_{1234} = \frac{3}{5}C.$$

Charge in the right branch of circuit (with capacitors $C_{1\ldots4}$) is $Q_{1234} = U \cdot C_{1234}$, which is the same as the charge of capacitor C_4 and also $C_{1\ldots3}$: $Q_{1234} = Q_3 = Q_{123}$. Voltage on $C_{1\ldots3}$ is $U_{123} = Q_{123}/C_{123}$. Capacitors C_1 and C_2 will have the same charge, $Q_1 = Q_2 = U_{123} \cdot C_{12} = U_{123} \cdot \frac{C}{2}$. Generally

$$Q_1 = \frac{1}{2}U_{123}C = \frac{1}{2}CU\frac{C_{1234}}{C_{123}} = \frac{1}{5}CU\,.$$

Plugging the numbers in, we get $Q_1 = 20\,\mathrm{\mu C}$.

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Problem FoL.15 ... glasses or contact lenses

Pepa needs glasses with focal length $f_\mathrm{b} = 50\,\mathrm{cm}$ *to clearly see his favourite crossword at a distance* $D = 25\,\mathrm{cm}$ *from his eye lens. The glasses are at* $d = 2\,\mathrm{cm}$ *from his eye lens which itself has focal length* $f_\mathrm{o} = 2\,\mathrm{cm}$. *What focal distance (in cetimetres) must Pepa's contact lenses – which are in direct contact with the eye – have so that Pepa still could see the crossword clearly without any change in the focal length of the eye lens? Treat all lenses as thin.*
Lukáš stared into crosswords.

First we calculate distance from the eye lens in which the image of the crossword is produced. Crossword is at a distance $d_\mathrm{b} = D - d$ from the glasses. This is less than f_b which means the image will be on the same side of the lens as the crossword. Image position with respect to the glasses is obtained from the lensmaker's equation

$$-s_\mathrm{b} = \frac{1}{\frac{1}{f_\mathrm{b}} - \frac{1}{D-d}}\,.$$

The image is then at a distance $s_\mathrm{o} = s_\mathrm{b} + d$ from the eye lens. From this directly follows final position of the image x:

$$x = \frac{1}{\frac{1}{f_\mathrm{o}} - \frac{1}{s_\mathrm{o}}} = \frac{1}{\frac{1}{f_\mathrm{o}} - \frac{f_\mathrm{b}-D+d}{d(f_\mathrm{b}-D+d)+f_\mathrm{b}(D-d)}} = \frac{1204}{575}\mathrm{cm}\,.$$

Focal distance f_c of the contact lens must fulfill

$$\frac{1}{f_\mathrm{c}}+\frac{1}{f_\mathrm{o}}=\frac{1}{x}+\frac{1}{D}\,,$$

which leads to

$$f_\mathrm{c}=\frac{1}{\frac{1}{D}-\frac{f_\mathrm{b}-D+d}{d(f_\mathrm{b}-D+d)+f_\mathrm{b}(D-d)}}\doteq 56.9\,\mathrm{cm}$$

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Problem FoL.16 ... superluminal electron

Assume the Bohr model of an ionized atom with fixed nucleus (with proton number Z) and only one electron. Which would be the lowest possible proton number of the atom in order to obtain superluminal velocity of the orbiting electron? Assume that the electron is in a ground state. Speed of light is $c=299.8\cdot 10^{6}\,\mathrm{m\cdot s^{-1}}$, charge of the electron is $e=-1.6022\cdot 10^{-19}\,\mathrm{C}$, Coulomb's constant is $k_\mathrm{e}=8.987\cdot 10^{9}\,\mathrm{N\cdot m^{2}\cdot C^{-2}}$ and the reduced Planck constant is $\hbar=1.0546\cdot 10^{-34}\,\mathrm{J\cdot s}$. Use classical, not relativistic, physics. *Jakub wanted to destroy the world.*

Velocity of the electron in the ground state can be obtained directly from the Bohr model

$$v_\mathrm{e}=\frac{Ze^2k_\mathrm{e}}{\hbar}\,.$$

We want this velocity to be superluminal, hence

$$v_\mathrm{e}=\frac{Ze^2k_\mathrm{e}}{\hbar}>c\,.$$

Playing around with this algebracic expression, we get condition for Z

$$Z>\frac{c\hbar}{e^2k_\mathrm{e}}\doteq 137.05\,.$$

Number of protons is an integer, so the velocity of the electron crossed the superluminal barrier for $Z=138$.

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Problem FoL.17 ... ecocar

Find the mass of a compressed gas which is equivalent to one litre of petrol with a heating value of $L=30\,\mathrm{MJ\cdot l^{-1}}$, given that we use air as our working gas with an initial pressure of $p_0=200p_\mathrm{a}$, where $p_\mathrm{atm}=10^{5}\,\mathrm{hPa}$ is the atmospheric pressure. Further assume that the density of air under atmospheric pressure is $\varrho_\mathrm{atm}=1.3\,\mathrm{kg\cdot m^{-3}}$, that it exhibits ideal-gas behaviour and that the expansion process is carried out isothermally without any loss of energy.
Lukáš came up with this on the trip in the mountains.

Let us write the ideal gas law for the original, intermediate and atmospheric pressure

$$p_0V_0=nRT\,,\quad pV=nRT\quad p_\mathrm{atm}V_\mathrm{atm}=nRT\,.$$

The temperature and the amount of substance are constant. The work done by gas during the expansion process is

$$W = \int_{V_0}^{V_\mathrm{atm}} p\mathrm{d}V = \int_{V_0}^{V_\mathrm{atm}} p_0V_0/V\mathrm{d}V = p_0V_0 \ln\frac{V_\mathrm{atm}}{V_0} = p_0V_0\ln\frac{p_0}{p_\mathrm{atm}}\,.$$

The density of air is $\varrho = m/V_\mathrm{atm}$, for which we can write

$$p_\mathrm{atm} = \frac{\varrho_\mathrm{atm}}{M_\mathrm{m}}RT\,,$$

where M_m is molecular mass of the air. By the ideal gas law for the initial state, we can express p_0V_0 as.

$$p_0V_0 = \frac{m}{M_\mathrm{m}}RT = m\frac{p_\mathrm{atm}}{\varrho_\mathrm{atm}}\,.$$

The work has already been computed before, so let us assume that it is equal to the heating value of one litre of petrol

$$LV_\mathrm{B} = W = p_0V_0\ln\frac{p_0}{p_\mathrm{atm}} = m\frac{p_\mathrm{atm}}{\varrho_\mathrm{atm}}\ln\frac{p_0}{p_\mathrm{atm}}\,,$$

whence

$$m = \frac{LV_\mathrm{B}\varrho_\mathrm{atm}}{p_\mathrm{atm}\ln\frac{p_0}{p_\mathrm{atm}}} \doteq 73.6\,\mathrm{kg}\,.$$

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Problem FoL.18 ... swinging barrel

A cylindrical object with a radius of $r = 0.5\,\mathrm{m}$, height $l = 3\,\mathrm{m}$ and mass of two tonnes is floating on the water of density ϱ so that the axis of the cylinder remains vertical. Let us displace the cylinder from its equilibrium position vertically by $\Delta x = 1\,\mathrm{mm}$. Find the period of oscillations of the cylinder (in seconds). The acceleration due to gravity is $g = 9.81\,\mathrm{m{\cdot}s^{-2}}$.

Pikoš's problem from the school days.

There is a force due to gravity $F_G = mg$ acting on the cylinder, where m is the mass and g is the acceleration due to gravity. The other force acting on the cylinder is the buoyant force

$$F_\mathrm{v} = V\varrho g\,,$$

where V is the volume of the submerged part of the cylinder and ϱ is the density of the liquid (water). At the equilibrium position, the resultant force acting on the cylinder is zero. Let us denote the height of the submerged part as x_0 (at the equilibrium position). Then $mg = \pi r^2 x_0\varrho g$. If we displace the cylinder from the equilibrium position by x upwards, then the height of the submerged part will be $x_0 - x$ and the magnitude of the resultant force will be equal to

$$F = mg - \pi r^2(x - x_0)\varrho g\,.$$

Substituting for x from the previous equation we obtain

$$F = -\pi r^2 x\varrho g\,,$$

thus the force acting on the cylinder is proportional to the displacement, hence we can see that it is a simple harmonic oscilator with the effective spring constant $\pi r^2 \varrho g$. Therefore, the sought-after period of oscillations is

$$T = 2\pi\sqrt{m/(\pi r^2 \varrho g)} \doteq 3.2\,\mathrm{s}\,.$$

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Problem FoL.19 ... capacitor duo

Find the charge in Coulombs on the capacitor C_2 if you know the following: At the beginning, the switch S_0 was off and the switch S_1 was on, as displayed on the picture. There was zero voltage on both capacitors. Then we switched S_0 on and waited until the current stopped flowing. Subsequently, we switched S_1 off and again waited for the circuit to come into a steady state. At the end, we measured the charge on the capacitor C_2. The voltage across the ideal voltage source (DC) is $U = 17\,\mathrm{V}$ and both capacitors have the same capacitance $C = 1\,\mu\mathrm{F}$.

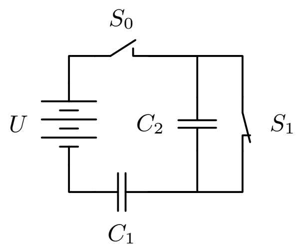

Fig. 2: To the problem 19

Náry felt like doing some electrotechnics.

This experiment represents a process in which we add an extra capacitor into a capacitor circuit in a steady state. This new element acts as a simple conductor because it is added at a moment when all differences in the electric potential are balanced. There is a zero voltage on the added capacitor and thus also a zero accumulated charge.

Validity of this statement can be seen from the second Kirchhoff's circuit law and the conservation of the electric charge.

$$\frac{Q+q}{C} + \frac{q}{C} = U\,, \tag{2}$$

where Q is the original charge on the positively charged plate of the capacitor C_1 and q is an extra charge on this plate after switching off S_1. This is then the charge lost by the plate of C_2 which is connected with the positively charged plate of C_1.

From the first phase of the process we know

$$\frac{Q}{C} = U\,, \tag{3}$$

which, combined with (2) gives $q = 0\,\mathrm{C}$.

Charge on the capacitor C_2 will be $0\,\mathrm{C}$.

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Problem FoL.20 ... heureka

Consider a cube with edge length $a = 1\,\mathrm{m}$ and density $\varrho_0 = 1\,000\,\mathrm{kg{\cdot}m^{-3}}$ and a container with a liquid of density (at its level) ϱ_0. The density of the liquid increases linearly with depth, so $\varrho(h) = \varrho_0 + \alpha h$, where $\alpha = 25\,\mathrm{kg{\cdot}m^{-4}}$. How deep does the cube sink, given that the height of the level of the liquid does not change having immersed the cube in the liquid? The acceleration due to gravity is $g = 9.81\,\mathrm{m{\cdot}s^{-2}}$. State your answer in centimetres.

Terka was bathing in strange liquids.

Let us use the Archimedes' principle: a thin slab of the cube with a height of $\mathrm{d}x$ submerged at a depth of x is acted on by a force corresponding to the weight $ga^2(\varrho_0 + \alpha x)\mathrm{d}x$ of the liquid of the same volume as the slab. The total mass of the water replaced by the cube's body must be equal to the total mass of the cube, so

$$\int_0^H (\varrho_0 + \alpha x)\mathrm{d}x = a^3 \varrho_0\,,$$

which leads to a quadratic equation. This can be solved as

$$0 = \alpha H^2 + 2\varrho_0 H - 2a^3 \varrho_0\,,$$

$$H = \frac{\sqrt{\varrho_0^2 + 2\alpha a^3 \varrho_0} - \varrho_0}{\alpha}\,.$$

Plugging in the numbers, we get $H \doteq 98.8\,\mathrm{cm}$.

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Problem FoL.21 ... ecology above all

Find the efficiency (energy returned on energy invested - ERoEI) of storing the energy in the form of compressed air. If there is surplus of electrical energy produced in solar power plants, we start to compress the air adiabatically so that we eventually achieve a compression ratio of $k = 10$. If we need the energy back, we let the air expand adiabatically to the original pressure. However, before we commenced the expansion process the air cooled down to the temperature before the compression took place. Assume that the air is a diatomic gas exhibiting ideal-gas properties. State your answer in terms of percentage.

Lukáš was listening to a programme about the cars running on compressed air.

Quantities indexed by 1 describe the initial state while the ones indexed by 2 describe the state after the adiabatic compression, 3 stands for the state before the adiabatic expansion and 4 is for the state after the expansion.

Let us derive the energy supplied to the gas during the adiabatic compression first (energy invested). We can write

$$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\kappa - 1} = T_1 k^{\kappa - 1}\,.$$

Then the energy invested is

$$E_{\text{in}} = RnT_1\left(k^{\kappa-1} - 1\right) .$$

For determining the parameters describing the state 3, the ideal gas law needs to be employed. We know that $T_3 = T_1$, hence

$$p_3 = kp_1 .$$

We should note that $p_4 = p_1$. Then we can write for the adiabatic expansion

$$T_4^{\kappa} = T_3^{\kappa}\left(\frac{p_3}{p_1}\right)^{1-\kappa} = T_1^{\kappa}k^{1-\kappa} \quad \Rightarrow \quad T_4 = T_1 k^{\frac{1-\kappa}{\kappa}} .$$

The energy returned in this process is

$$E_{\text{out}} = RnT_1\left(1 - k^{\frac{1-\kappa}{\kappa}}\right) .$$

At this point, we can compute the efficiency as

$$\eta = \frac{E_{\text{out}}}{E_{\text{in}}} = \frac{1 - k^{\frac{1-\kappa}{\kappa}}}{k^{\kappa-1} - 1} .$$

Let us recall that for a diatomic gas we have $\kappa = 1.4$, so the answer is $\eta \doteq 31.9\,\%$.

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Problem FoL.22 ... resistor blackbox

Consider a box containing three resistors with unknown resistances. These resistors are connected through ideal conductors. There are four output terminals leading outside the box. We measure the resistance across different pairs of terminals for all possible combinations of terminals. Five such measurements yield $9\,\Omega$, $12\,\Omega$, $8\,\Omega$, $3\,\Omega$ and $14\,\Omega$. Let us now connect the ohmmeter across the last unmeasured pair of terminals. What resistance do we find?

Náry is tragic.

Unless we want to measure zero resistance, there should not be any loop in the wiring, hence the resistors must be connected either in a star configuration or in series. Equipped with this condition we can uniquely determine both the precise way of wiring as well as the individual values of resistances of the resistors and hence the resistance across the last choice of terminals.

Let us denote the resistances by R_1, R_2 and R_3. Based on how these resistors may be connected we infer that the resistances measured across the terminals can either be a combination of sums of R_1, R_2 and R_3 or directly R_1, R_2 or R_3. There are exactly 7 such cases. Since we obtained our measurements for 5 possible combinations of terminals out of 6, we must have necessarily measured at least two values which correspond directly to some of R_1, R_2 and R_3. This implies that the least resistance measured across a pair of terminals must be one of R_1, R_2 or R_3, since assuming the opposite we come to a quick contradiction. Denoting the least resistance R_1, we observe that $R_1 = 3\,\Omega$.

In the next step we subtract R_1 from all of the remaining measured resistances. There exist exactly 3 pairs of resistors, which we potentially could have measured and which satisfy the following: when we subtract R_1 from the resistance of the first one we get the resistance of the second one. Since we conducted 5 measurements, there must be at least one such a pair in the list of measured resistances. Indeed, the values $12\,\Omega$ and $9\,\Omega$ differ exactly by $3\,\Omega$. Since this is the only such pair and simultaneously $12\,\Omega$ is not the highest value measured, we must have that $9\,\Omega$ is the resistance of the second resistor, denoted by R_2 (think this carefully through!).

Since $8\,\Omega$ is less than R_2, it must be equal either to R_3 or $R_1 + R_3$. However, it would be impossible to place the terminals in a star or serial wiring in such a way so as to obtain a resistance of $14\,\Omega$, if we had $R_3 = 8\,\Omega$. Hence $R_3 = R_2 - R_1 = 5\,\Omega$.

Based on the above derived results we easily conclude that the resistors are connected in a star and that the resistance across the last choice of terminals is $R_3 = 5\,\Omega$.

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Problem FoL.23 ... orbiters

Determine the magnitude of a magnetic field which is present at the centre of mass of the system of two planets charged with equal charges of $Q = 100\,\mathrm{TC}$ *and with equal masses of* $M = 5\cdot 10^{24}\,\mathrm{kg}$ *orbiting each other at a distance of* $d = 500\,000\,\mathrm{km}$ *apart. In your calculations, regard the two planets as point charges and assume that their orbits are circular. State your answer in terms of* nT. *Lukáš wanted to create an unconventional problem no matter what.*

The planets exert an attractive force on each other, which is of magnitude

$$F = \frac{1}{d^2}\left(GM^2 - \frac{Q^2}{4\pi\varepsilon_0}\right).$$

In order to balance the centrifugal with the attractive force (from the point of view of the frame of reference rotating with the planets), we have to satisfy

$$v = \sqrt{\frac{dF}{2M}}.$$

This means that the current flowing around the centre of mass will be

$$I = \frac{2Qv}{\pi d}$$

which allows us to compute the magnitude of the magnetic field at the centre of mass as

$$B = \mu_0 \frac{I}{\pi d} = \mu_0 \frac{2Qv}{(\pi d)^2} = \mu_0 \frac{2Q\sqrt{\frac{dF}{2M}}}{(\pi d)^2} \doteq 57.23\,\mathrm{nT}.$$

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Problem FoL.24 ... mercujet

Consider a flask filled with a potion of density $\varrho = 13\,600\,\mathrm{kg{\cdot}m^{-3}}$. The flask is sealed at the top, with a small capillary inserted in the seal. The potion is heated to the temperature corresponding to its boiling point under the atmospheric pressure $p_\mathrm{a} = 101\,325\,\mathrm{Pa}$. By how much will the level of the potion in the capillary rise? Neglect both the change in the surface tension with the temperature as well as the thermal expansion of both the potion and the flask. The acceleration due to gravity is $g = 9.81\,\mathrm{m{\cdot}s^{-2}}$. The flask is placed in vacuum. Assume that at the beginning of the process, the partial pressure of the potion vapor is zero.

Lukáš was playing with a PET bottle.

If we heat the potion up to temperature corresponding to its boiling point under the atmospheric pressure, the partial pressure of its vapor is the same as the atmospheric pressure, hence the potion will rise to a height of

$$h = \frac{p_\mathrm{a}}{\varrho g} \doteq 759\,\mathrm{mm}\,,$$

because the pressure above the level of the potion in the flask is p_a and there is zero pressure above the level of the potion in the capillary.

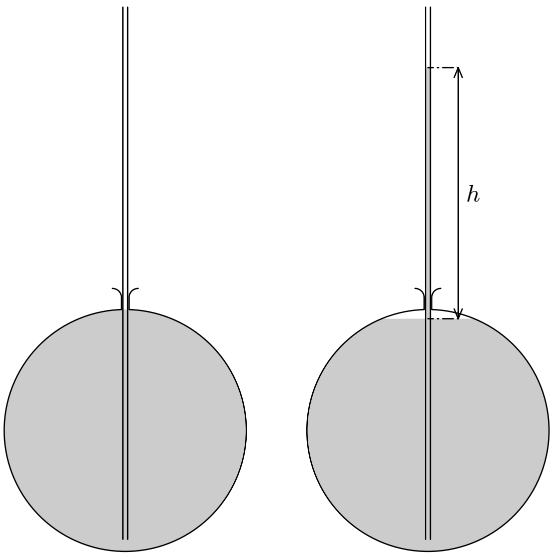

Fig. 3: english label 24

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Problem FoL.25 ... dimensionless hydrogen

Consider the ground state energy of the hydrogen atom (a system consisting of a proton and an electron) in the non-relativistic model with the proton fixed at a point. If the electron is *in the infinite distance from the proton, the energy of the configuration is defined to be zero. Find the ground state energy in terms of dimensionless units defined by putting the mass of the electron $m_e = 1$, reduced Planck constant $\hbar = 1$ and $k_e e^2 = 1$, where k_e is the Coulomb's constant and e denotes the electron's charge.* *Jakub was forced to think*

Considering the Bohr's model of atom, the energy of the ground state of hydrogen is

$$E_0 = -\frac{m_e e^4 k_e^2}{2\hbar^2}\,.$$

We simply substitute 1 for the quantities mentioned in the task to obtain the energy in dimensionless units

$$E_0 = -\frac{1}{2}\frac{(m_e)\left(k_e e^2\right)^2}{(\hbar)^2} = -\frac{1}{2}\,.$$

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Problem FoL.26 ... jumping dog

Imagine a marble placed in a height of $h = 1\,\mathrm{m}$ above the ground. At some point, we release the marble and simultaneously start to push the ground towards the marble with a speed of $V = 1.2\,\mathrm{m\cdot s^{-1}}$ with respect to the initial rest frame of the system. Given that the coefficient of restitution between the marble and the ground is $e = 0.3$, find the time needed for the marble to steady on the moving ground. We define the coefficient of restitution as the ratio of speeds of the marble with respect to the ground before and after each collision.

Kuba played with a marble.

Let us look at the situation from the inertial frame of reference connected with the moving ground. The initial speed of the marble in this frame is V. The marble will hit the surface at the time

$$t_1 = \sqrt{2\frac{h}{g} + \left(\frac{V}{g}\right)^2} - V/g$$

having a speed of

$$v_1 = V + gt_1 = \sqrt{2gh + V^2}\,.$$

Let us take into account the coefficient of restitution e. After the collision the marble will have a speed of $v_2 = ev_1$ and with this one, it will hit the surface again at the time

$$t_2 = 2\frac{v_2}{g} = 2e\frac{v_1}{g}\,,$$

subsequently hitting the surface again with a speed of $v_3 = ev_2$ etc. By induction, write

$$\forall n \in \mathbb{N}\backslash\{1\} : t_n = 2e^{(n-1)}\frac{v_1}{g}\,,$$

where t_1 is given above. If $e < 1$ the series $\sum t_n$ converges. Let T be the time we are looking for. Then

$$T = \sum_{n=1}^{\infty} t_n = t_1 + 2\frac{v_1}{g}\sum_{n=2}^{\infty} e^{(n-1)} = t_1 + 2\frac{v_1}{g}\sum_{n=1}^{\infty} e^n\,,$$

giving

$$T = t_1 + 2\frac{v_1}{g}\frac{e}{1-e} = \frac{\sqrt{2gh+V^2}}{g}\frac{1+e}{1-e} - V/g\,.$$

Plugging in the numbers, we get $T \doteq 0.74\,\mathrm{s}$.

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Problem FoL.27 ... P5

What is the average number of photons arriving from Pluto's spherical moon P5 onto the mirror of the Hubble Space Telescope every second? Assume the following: the moon has a diameter of $D = 20\,\mathrm{km}$, its distance from the Sun is $L = 32\,\mathrm{AU}$ and it has an albedo of $a = 0.3$. The Hubble Space Telescope is equipped with a mirror of diameter $d = 2\,\mathrm{m}$. You can regard the Sun as being a monochromatic light source, emitting at a wavelength of $\lambda = 550\,\mathrm{nm}$ with the solar constant being $P_\mathrm{S} = 1\,400\,\mathrm{W{\cdot}m^{-2}}$. Further assume that the Hubble Space Telescope is also located at a distance of L from P5. Do not consider the absorption in the interplanetary medium, assume the isotropic scattering of photons by P5 and do not take into account the photons absorbed and subsequently radiated back by P5.

Lukáš read some stuff about exoplanets.

Let us compute the power that the moon receives from the Sun

$$P_\mathrm{m} = P_\mathrm{S}\frac{1\,\mathrm{AU}^2}{L^2}\cdot\frac{\pi}{4}D^2\,.$$

This power (reduced by the albedo) is uniformly scattered by the moon onto a sphere with a radius of L and an area of $4\pi L^2$. However, we detect only that part corresponding to the area of the telescope's mirror being $S_\mathrm{d} = \pi d^2/4$. Hence the power detected is

$$P_\mathrm{o} = aP_\mathrm{m}\frac{\pi d^2/4}{4\pi L^2} = P_\mathrm{S}a\frac{1\,\mathrm{AU}^2}{L^2}\cdot\frac{\pi}{4}D^2\frac{\pi d^2/4}{4\pi L^2} = P_\mathrm{S}a\frac{\pi}{64}\frac{1\,\mathrm{AU}^2D^2d^2}{L^4} \doteq 1.398\cdot 10^{-18}\,\mathrm{W}\,.$$

Further we have to determine the energy of one photon of given wavelength. It is true that $E = hc/\lambda = 3.638\cdot 10^{-19}\,\mathrm{J}$. Hence, for the number of photons detected every second we have $P_\mathrm{o}/E = 3.842\,\mathrm{Bq}$, where Bq is a unit with the same dimension as Hz. Though this one is used for random processes, while the latter is for periodic ones. The telescope will receive 3.8 photons per second on average.

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Problem FoL.28 ... weird gravity

Imagine that you are standing on the inner side of the wall of a huge open-ended cylinder with a base radius of $R = 1000\,\mathrm{m}$, The cylinder is rotating about its axis with a constant angular speed so that the centrifugal acceleration you can feel is the same as the acceleration due to gravity on the Earth's surface $g = 9.81\,\mathrm{m{\cdot}s^{-2}}$. The cylinder itself is virtually massless and it is placed in vacuum, outside the influence of gravitating bodies. Imagine you throw a ball straight up from the surface you are standing on, giving it an initial speed of $v = 10\,\mathrm{m{\cdot}s^{-1}}$ in *a direction perpendicular to the surface. Compute how far from you the ball lands. The distance is measured along the surface, in the frame of reference connected with the rotating cylinder. State your answer in metres.* *Kuba whirling around.*

If there is to be a normal acceleration of g on the inner side of the wall, the angular speed must be $\omega = \sqrt{g/R}$. We need to remember that there are no forces acting on the flying ball in the non-rotating reference frame which is thus inertial, so in this frame the ball is either at rest or it moves in a straight line with constant speed. In fact, in such frame the ball's trajectory will be a straight line which makes an angle α with the direction normal to the surface, where

$$\operatorname{tg}\alpha = \frac{\sqrt{gR}}{v}.$$

The speed of the ball in this frame is

$$v' = \sqrt{v^2 + gR},$$

so the straight trajectory of the ball hits the wall of the cylinder at time

$$t = \frac{2R\cos\alpha}{\sqrt{v^2+gR}} = \frac{2Rv}{v^2+gR}.$$

But during this time, the cylinder rotates through an angle of ωt, so the distance to the point of landing measured along the surface is

$$d = R(\pi - 2\alpha - \omega t) = R\left(\pi - 2\operatorname{arctg}\frac{\sqrt{gR}}{v} - \frac{2Rv}{v^2+gR}\sqrt{\frac{g}{R}}\right).$$

Plugging in the numbers, $d \doteq 1.36\,\mathrm{m}$.

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Problem FoL.29 ... gramme of hexagram

Consider a fractal which resulted from infinite plunging of hexagrams into each other (see the figure). Only the coloured parts have mass and their total mass is exactly one gramme. The massive parts are made of homogeneous material and the radius of the fractal is one centimetre (the radius is measured from the centre to the furthermost vertex of the fractal). What is the moment of inertia with respect to the axis perpendicular to the plane of the fractal and going through the centre of the fractal? The result should be stated in terms of $\mathrm{g{\cdot}cm^2}$*. The moment of inertia of an equilateral triangle with a side lenght of* a_t *and mass* m_t *with respect to the axis perpendicular to the plane of the triangle and going through its centre of mass is*

$$I_\mathrm{t} = \frac{1}{12} m a_\mathrm{t}^2\,.$$

Karel was thinking about moments of inertia.

Let the radius of the fractal be r. By looking at the geometry of the problem, we note that the ratio of the dimensions of every inner and outer triangle is $q = 1/\sqrt{3}$. Let us denote the area of the coloured part depicted on the first diagram by S_1, the area of the coloured part which

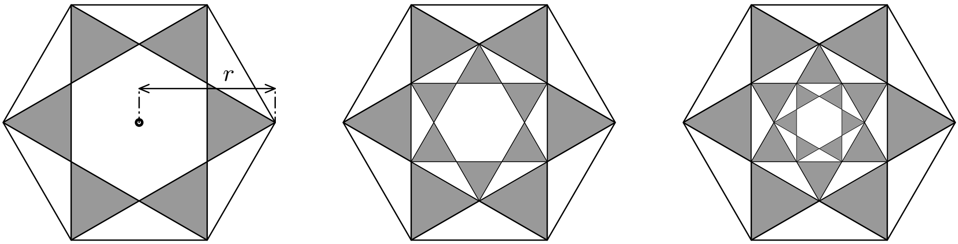

Fig. 4: To the problem 29

was added in the the second diagram will be denoted by S_2 etc. The area is proportional to the square of the linear dimensions of a tringle, so

$$S_{n+1} = q^2 S_n \,.$$

The total area is then

$$S = \sum_{n=1}^{\infty} S_n = S_1 \frac{1}{1-q^2} = \frac{3}{2} S_1 \,.$$

The lengths of the edges of the equilateral triangles in the first diagram are $a_1 = r/\sqrt{3}$, so the area of the whole fractal is

$$S = \frac{3}{2} S_1 = \frac{3}{2} \cdot 6 a_1^2 \frac{\sqrt{3}}{4} = r^2 \frac{3\sqrt{3}}{4} \,.$$

Hence the surface density is

$$\varrho = \frac{m}{S} = \frac{4}{3\sqrt{3}} \frac{m}{r^2} \,.$$

The moment of inertia of the shape depicted on the first diagram, I_1, is six times the moment of inertia of an equilateral triangle with respect to the axis going through the centre of the fractal. To compute this moment of inertia, we need to employ the parallel axis theorem

$$\frac{I_1}{6} = I_{\mathrm{t1}} + m_{\mathrm{t1}} \cdot l_{\mathrm{t1}}^2,$$

where I_{t1} is the moment of inertia of the triangle with respect to the axis going through its centre of mass, m_{t1} is its mass and l_{t1} is the distance between the centre of mass of the triangle and the centre of the fractal. We know the dimensions of the triangles as well as their surface density and $l_{\mathrm{t1}} = \frac{2}{3} r$. From the geometry of the problem, we can write

$$I_1 = 6 \left(I_{\mathrm{t1}} + m_{\mathrm{t1}} \cdot l_{\mathrm{t1}}^2 \right) = \frac{17}{54} m r^2 \,.$$

If the surface density is constant, the moment of inertia is proportional to the fourth power of the dimension ($I \propto mr^2$ and $m \propto r^2$), so for the plunged stars

$$I_{n+1} = q^4 I_n \,.$$

Summing the resulting geometrical progression, we get a finite moment of inertia

$$I = \sum_{n=1}^{\infty} I_n = I_1 \frac{1}{1-q^4} = \frac{9}{8} I_1 = \frac{17}{48} m r^2 \,.$$

Substituting the numerical values, we get $I = 0.354\,\mathrm{g{\cdot}cm^2}$.

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Problem FoL.30 ... crystal mirror

Consider an aquarium suspended in the air. We can assume that it is infinitely big, so no matter from which direction the rays are coming, they always hit a wall (see the figure). The walls are made of glass with a refractive index of $n_1 > 1$. The aquarium is filled with an unknown transparent liquid with a refractive index of $n_2 > 1$. The refractive index of air is $n = 1$. There is a sheet of paper placed under the aquarium with the solution to this problem written down on it. Find the smallest refractive index of the liquid so that we would not be able to see the paper from aside of the aquarium (see the figure).

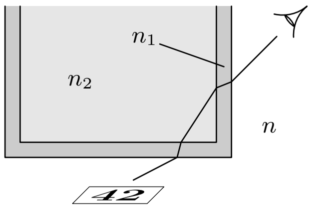

Fig. 5: To the problem 30

f(Aleš) brought up a problem during the brainstorming.

The light is passing through several layers with different refractive indices. For such a situation, we write

$$n_1 \sin\varphi_1 = n_2 \sin\varphi_2 = \cdots = n_N \sin\varphi_N \,. \tag{4}$$

Hence when passing through the aquarium wall for the first time, the rays refract independently on the refractive index of the glass and the direction in which they are deflected is affected by the refractive index of the liquid only. Since we have $n_1 > n$, the rays always pass into the aquarium.

The rays can hit the bottom of the aquarium under a range of angles of $(0, \pi/2)$. We see from the equation (4) that should the total internal reflection occur, it will certainly happen no later than at the last two interfaces. In other words, if there is to be a total internal reflection on the interfaces liquid–glass or glass-air, it would occur on the interface liquid–air as well. Hence, it is true that

$$n \sin\frac{\pi}{2} = n_2 \sin\varphi_2 \,,$$

whence

$$n_2 = \frac{1}{\sin\varphi_2} \,,$$

where φ_2 is the angle of incidence of the rays passing through the liquid onto the glass. From the symmetry of the problem, by considering the rays travelling in the opposite direction, we can write $\varphi_2 = \pi/4$. Hence we end up with a condition

$$n_2 \geq \sqrt{2}\,,$$

thus $n_2 = \sqrt{2}$, which is approximately 1.41.

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Problem FoL.31 ... running mirror

Consider a system containing a converging lens with a focal lenght of $f = 20\,\mathrm{cm}$ and a movable convex mirror with a radius of curvature of $3f$ (see the figure). At $t = 0$, when the mirror and the lens are in contact, we start to move the mirror with a speed of $v = 1\,\mathrm{m{\cdot}s^{-1}}$ away from the lens. What should be the position of the object as a function of time, in order for its image to stay at a distance of $2f$ leftwards from the lens? Assume that you can write $x(t) = f \cdot (v^2t^2 + vft - 3f^2)/(v^2t^2 - A)$ for the position of the object. Determine the constant A. Assume that the speed of light is infinite and that you can use the paraxial approximation.

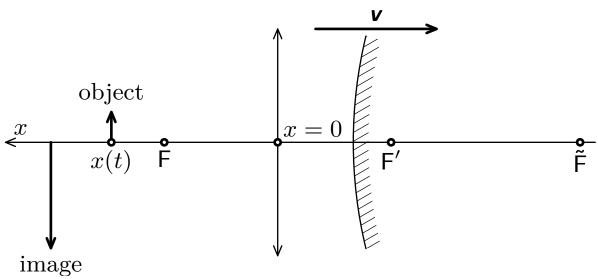

Fig. 6: To the problem 31

Lukáš, sitting on an optical bench.

Let us denote the position of the object by x, the position of the image produced by the lens by x_c, the position of the image after imaging by the mirror by x_cz and the position after the final imaging by the lens by x_czc. Further denote by d the distance of the mirror from the lens. Then we can write

$$\frac{1}{x} + \frac{1}{x_\mathrm{c}} = \frac{1}{f}\,, \tag{5}$$

$$\frac{1}{d - x_\mathrm{c}} + \frac{1}{d - x_\mathrm{cz}} = \frac{2}{r}\,, \tag{6}$$

$$\frac{1}{x_\mathrm{cz}} + \frac{1}{x_\mathrm{czc}} = \frac{1}{f}\,, \tag{7}$$

Further we have to substitute $x_\mathrm{czc} = 2f$ and $r = -3f$ due to the standard sign convention. Without these two assumptions, we can write the position of the object as a function of the

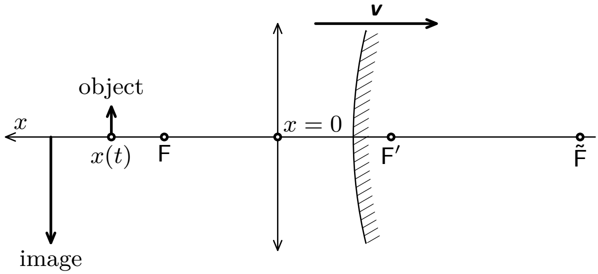

Fig. 7: To the problem 31

position of its final image and the distance of the mirror from the lense, which yields somewhat complicated expression. We then have

$$x = \frac{d^2 f + df^2 - 3f^3}{d^2 - \frac{5}{2}f^2}\,,$$

whence, substituting $d = vt$

$$x(t) = f \cdot \frac{v^2t^2 + vft - 3f^2}{v^2t^2 - \frac{5}{2}f^2}\,.$$

We observe that $A = 5f^2/2 = 0.1\,\mathrm{m}^2$.

This approach to the problem is somewhat technical and we need not have followed this path on our way to the final solution, since it is sufficient to determine the position of the mirror for one particular position of the object.

For this purpose we choose $x = f$. Then, by (5), we have $x_{\mathrm{c}} = \infty$. Forther we know that $x_{\mathrm{czc}} = 2f$ and by (7) we get $x_{\mathrm{cz}} = 2f$, hence for d by (6) we have

$$\frac{1}{d-\infty} + \frac{1}{d+f} = -\frac{2}{3f} \quad\Rightarrow\quad \frac{1}{d-2f} = -\frac{2}{3f} \quad\Rightarrow\quad d = \frac{1}{2}f\,.$$

This situation occurs at $t = d/v = f/(2v)$. We substituted in to the ansatz given in the task and having divided by f, we obtain

$$1 = \frac{v^2t^2 + vft - 3f^2}{v^2t^2 - A} \quad\Rightarrow\quad v^2t^2 - A = v^2t^2 + vft - 3f^2 \quad\Rightarrow\quad A = \frac{5}{2}f^2 = 0.1\,\mathrm{m}^2\,.$$

The numerical value of the sought-after constant A je $0.1\,\mathrm{m}^2$.

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Problem FoL.32 ... charged robber

A robber weighing $m = 50\,\mathrm{kg}$ was running away from the policemen. He decided to save himself by jumping from a cliff. Luckily enough, he stole a charge of $q = 10\,\mathrm{C}$. There is a homogeneous magnetic field below the cliff, reaching the height $a = 10\,\mathrm{m}$. The field is perpendicular to the vertical and its magnitude is $B = 10\,\mathrm{T}$. What is the maximum height of the cliff h (in metres) *from which the robber can jump without reaching the surface? Do not take into account the air resistance. The acceleration due to gravity is* $9.81\,\mathrm{m\cdot s^{-2}}$ *and the velocity of the robber at the moment of entering the magnetic field has a vertical component only. Assume that the magnetic field is pointing in the right direction so the robber does not hit the cliff.*

From the head of Tomáš B.

Let us assume that the cliff is on the right hand side, so we chose a right-handed cartesian coordinates system so that the x axis points to the left and y axis is points down. Let the $y = 0$ plane be the boundary of the region with the magnetic field pointing in the direction of z. Employing the Lorentz force, the equations of motion can be written as

$$m\ddot{x} = qB\dot{y}\,,$$
$$m\ddot{y} = -qB\dot{x} + mg\,.$$

Let us start to measure the time at the moment when the robber enters the magnetic field. At this moment we have $\dot{x}(0) = 0$ and $y(0) = 0$. After integrating the first equation and substituting into the second one, we get

$$\ddot{y} = -\left(\frac{qB}{m}\right)^2 y + g\,.$$

The vertical motion of the robber can apparently be described by the equation of simple harmonic oscilator, so

$$\frac{1}{2}m\dot{y}^2(0) + \frac{1}{2}\frac{q^2B^2}{m}\delta^2 = \frac{1}{2}\frac{q^2B^2}{m}(a-\delta)^2$$

whence we can obtain

$$h = \frac{q^2B^2a^2}{2gm^2} \doteq 20.4\,\mathrm{m}\,.$$

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Problem M.1 ... drive safely!

Two cars with different types of tyres are decelerating in summer on a straight dry road from a velocity $v_0 = 100\,\mathrm{km{\cdot}h^{-1}}$. Initially, they are riding next to each other and start braking at the same time. When the car with summer tyres comes to the rest the other one, with winter tyres, is still moving with a velocity $v_1 = 37\,\mathrm{km{\cdot}h^{-1}}$ and stops after 6 more meters. What is the fraction of the horizontal deceleration of the car with winter tyres to the horizontal deceleration of the other car? *Michal heard this on radio.*

Let us denote decelerations of the cars with winter and summer tyres a_w and a_s respectively. It obviously holds

$$v_0 - a_w(v_0/a_s) = v_1\,.$$

Thus

$$(1 - a_w/a_s) = v_1/v_0\,,$$

which allows us to obtain the desired fraction $a_w/a_s \doteq 0.63$.

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Problem M.2 ... wheelspokes

Consider an eight-spoked wheel with radius $r = 30\,\mathrm{cm}$ rotating with angular speed $\omega = 2.5\pi\,\mathrm{rad{\cdot}s^{-1}}$ about a fixed axle through its centre. There are some boys shooting a bow in the direction of the wheel, trying to make the arrows pass freely through the gaps between the spokes. The length of one arrow is $l = 23\,\mathrm{cm}$. Assuming that the spokes and the arrows are negligibly thin, find the minimal speed of the arrows so that the boys would succeed in their objective. State your answer in metres per second. *Zdeněk and his head spinning all around.*

Let us write the frequency in terms of the angular angular speed

$$f = \frac{\omega}{2\pi}\,.$$

For the arrow not to be hit by a moving spoke, it must pass through the wheel in less than one eigth of the period T, where

$$\frac{T}{8} = \frac{1}{8f}\,.$$

This must be equal to the time which it takes the arrow to travel through the distance equal to its length, thus

$$v = 8fl \doteq 2.3\,\mathrm{m{\cdot}s^{-1}}\,.$$

Note that the information about the radius is completely redundant, unless we consider the spokes and arrows to be of finite thickness.

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Problem M.3 ... spleen on a bridge

Bored person is standing on a bridge with the height $h = 15\,\mathrm{m}$ *and is dropping pebbles on cars passing along a straight road beneath him. In a distance he spots an approaching motorcyclist and decides to hit him. He estimates the motorcyclists's instantaneous velocity to be* $v = 72\,\mathrm{km{\cdot}h^{-1}}$ *and his horizontal distance* $d = 500\,\mathrm{m}$*. He calculates when he should drop the pebble and he indeed drops it at the calculated moment. However, when the pebble hits the ground the motorcyclist is already* $x = 50\,\mathrm{m}$ *behind the intended point of collision. What is the difference, in kilometers per hour, between the estimate of the motorcyclist's velocity and his actual velocity if we assume that the initial horizontal distance is guessed precisely and the velocity was constant throughout the motion?* *Kiki, during a stroll in Brno.*

Legthy instructions are compensated by an easy and quick solution. The time t when the pebble hits the ground is given by $t = d/v$. If the motorcyclist already drove a distance $s = d + x$ at that time, then his actual velocity was $v_\mathrm{a} = s/t$. Therefore it holds

$$v_\mathrm{a} = \frac{(d+x)}{d/v}\,.$$

After plugging in numbers in appropriate units we substract the estimated velocity and evaluate their difference $\Delta v = v_\mathrm{a} - v$ which is numerically $7.2\,\mathrm{km{\cdot}h^{-1}}$.

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Problem M.4 ... broken altimeter

A curious sky observer notices an airplane which is approaching in a way that it eventually passes exactly above his head. At one point, when the airplane is still approaching, the observer measures that it is $\alpha_1 = 1.3\,\mathrm{rad}$ *above the horizon. However, a noise from its engines is coming from a direction* $\alpha_2 = 0.5\,\mathrm{rad}$ *above the horizon. The observer measures an angular velocity of the airplane at the moment when it is passing above his head* $\omega = 0.125\,\mathrm{rad{\cdot}s^{-1}}$*. Based on these inputs calculate the height* h *of the airplane, assuming it is constant throughout the motion. The speed of sound is* $c = 340\,\mathrm{m{\cdot}s^{-1}}$ *and unvarying with the altitude. Neglect the finite speed of light.* *Kuba was crossing half of the globe.*

We assume that the airplane is travelling parallelly with the surface of Earth. At the moment when the airplane is passing above the observer we can write

$$v = \omega \cdot h,$$

where v is a velocity of the airplane.

Sound which the observer hears under the angle α_2 is coming from the distance

$$d = \frac{h}{\sin\alpha_2}\,.$$

And the distance between point which the observer sees under the angle α_1 and the point from which he hears the sound is

$$s = h\left(\frac{1}{\operatorname{tg}\alpha_2} - \frac{1}{\operatorname{tg}\alpha_1}\right)\,.$$

The airplane travelled the distance s in the same time as the sound covered the distance d. Therefore it holds

$$\frac{d}{c} = \frac{s}{v} = \frac{1}{\omega}\left(\frac{1}{\operatorname{tg}\alpha_2} - \frac{1}{\operatorname{tg}\alpha_1}\right) .$$

Substituion for d from the equation above finally yields

$$h = \frac{c\sin\alpha_2}{\omega}\left(\frac{1}{\operatorname{tg}\alpha_2} - \frac{1}{\operatorname{tg}\alpha_1}\right) .$$

And after numerical evaluation we get $h \doteq 2\,025\,\mathrm{m}$.

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Problem E.1 ... a different one

Filip, who is colour blind and equipped with a red He–Ne *laser (providing light with wavelength* $\lambda_1 = 633\,\mathrm{nm}$*), decided to measure the refractive index of his little glass made of borosilicate glass (designated as BSC7) for the wavelength of his laser (corresponding to red colour). The method used was to measure the critical angle of refraction of the laser beam incident on glass-vacuum interface and to infer the refractive index thereof. However, because of well-sustained mess in the container where he stores his lasers, he used a green laser (wavelength* $\lambda_2 = 555\,\mathrm{nm}$*) instead of the red one, by mistake. Find the fractional error of his result in terms of permille (parts per thousand) assuming that the measurement was not subject to other errors of any kind. For the BSC7 glass we have the respective refractive indices for the wavelengths* $\lambda_1 = 633\,\mathrm{nm}$ *and* $\lambda_2 = 555\,\mathrm{nm}$ *equal to* $n_1 = 1.51508$ *and* $n_2 = 1.51827$.

Honza stumbled upon while in optics lab.

It does not take one too long to find out that based on the measurement strategy described in the task, it is possible to find the refractive index for given wavelength directly, since $\sin\alpha_c = \frac{1}{n}$, where α_c is the critical angle and n is the sought-after refractive index. Hence, in order to find the answer we only need to know the refractive indices for given wavelengths in given material. The answer then reads

$$p = \frac{n_2 - n_1}{n_1} \doteq 2.1\,‰\,.$$

Jan Česal

Problem E.2 ... firefly

A neon lamp is connected through a resistor to a rigid source of alternating voltage of a root mean squre voltage 230 V *and frequency* 50 Hz*. Its ignition voltage (the striking voltage) is* 120 V *and the maintaining voltage is* 80 V*. How long it will stay lit during one half-period? Assume that all resistors in the circuit are such that you do not have to take drop in the current into consideration. Please provide the result in* ms*.*

f(Aleš) wanted to read in the evening but he didn't have any lamp.

Time dependence of a voltage is given by $u(t) = U\sin(\omega t)$. The root mean square voltage is defined by $U_{\mathrm{rms}} = U\sqrt{2}$. Therefore we can write for the ignition voltage

$$U_{\mathrm{I}} = \sqrt{2}U_{\mathrm{rms}}\sin(\omega t_1)\,,$$

where t_1 is the time when the lamp lits, if we start measuring time when the voltage is zero. If we express the time t_1

$$t_1 = \frac{\arcsin\left(\frac{U_{\mathrm{I}}}{\sqrt{2}U_{\mathrm{rms}}}\right)}{\omega}\,.$$

Following the same procedure we get time t_2 when the neon lamp goes out

$$t_2 = \frac{\arcsin\left(\frac{U_{\mathrm{M}}}{\sqrt{2}U_{\mathrm{rms}}}\right)}{\omega}\,,$$

where U_{M} is the maintaining voltage. We also put into use a familiar expression $\omega = 2\pi f$.

For both times we get two results. In the case of the time t_1 we are interested in the lesser one and in the case of the time t_2 in the bigger one in order to determine whole time when the neon lamp stays lit. Desired answer is then $t = t_2 - t_1$. Numerically we get

$$t_1 \doteq 1.20\,\mathrm{ms}\,,$$
$$t_2 \doteq 9.21\,\mathrm{ms}\,,$$

and thus

$$t \doteq 8.01\,\mathrm{ms}\,.$$

Finally we round the answer to 8 ms.

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Problem E.3 ... shut it down!

A square elevator with mass $m = 1000\,\mathrm{kg}$ and a side length of $a = 3\,\mathrm{m}$ is moving in a very long square shaft of a side length $b = 4\,\mathrm{m}$ with a speed of $v = 2\,\mathrm{m{\cdot}s^{-1}}$. Since this is the experimental physics building the elevator has a point charge $q = 1\,\mathrm{C}$ embedded in the middle of its floor. The most problematic lab just created a strong homogenous electric field with potential difference between the walls of the shaft $U = 1000\,\mathrm{V}$. The electric field is perpendicular to the motion of the elevator and also perpendicular to the walls of the shaft. The suspension of the elevator is so long that you can assumme that it moves along a straight line. What is the maximal time for which the electric field can last so the elevator still does not hit the wall of the shaft during the field's action?
f(Aleš) had an afternoon filled with thoughts about elevators and electricity kept meddling into it.

The elevator is subject to the electric force $F_\mathrm{e} = qE$ with electric intensity given by $E = U/b$. The acceleration in the perpendicular direction is

$$a = \frac{qU}{mb}\,.$$

The elevator cannot move in the perpendicular direction further than $s = (b - a)/2$ which will take time

$$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2\left(\frac{b-a}{2}\right)mb}{qU}} = \sqrt{\frac{(b-a)\,mb}{qU}}\,.$$

Numerically, we get $t = \sqrt{4}\,\mathrm{s} = 2\,\mathrm{s}$.

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Problem E.4 ... anullator

Consider two long parallel conductors $\{a\}^{\{a\}}$ metres apart, where $\{a\}$ denotes the numerical value of a physical quantity a, so $a = \{a\}[a]$, where we write $[a]$ for the unit of a. Both conductors are placed in vacuum and the currents through them flow in the directions opposite to one another. Assume that a current of $I_1 = 1\,\mathrm{A}$ flows through the conductor nr. 1. Find the current *flowing through the conductor nr. 2 given that the magnetic field is zero at a perpendicular distance of* $\{b\}^{\{b\}}$ *from the conductor nr. 1 (that one which is further from the conductor nr. 2). Further assume that the distance* a *is* $30\,277\,604\,100\,\mathrm{m}$ *longer than one astronomical unit, that the light travels through the distance* b *in 10 minutes and that the astronomical unit is precisely* $149\,597\,870\,700\,\mathrm{m}$. *f(Aleš) wrote a sweet dot.*

The magnitude of a magnetic field $\boldsymbol{B}$ at a distance of r from a conductor can be written as

$$B = \frac{\mu}{2\pi}\frac{I}{r}\,.$$

We need to have $\boldsymbol{B}_1 + \boldsymbol{B}_2 = 0$, so

$$\frac{\mu}{2\pi}\left(\frac{I_1}{r_1} - \frac{I_2}{r_2}\right) = 0\,,$$

whence

$$|I_2| = \left|I_1\frac{r_2}{r_1}\right|\,.$$

Now, let us remember that $r_2 = \{a\}^{\{a\}}\,\mathrm{m} + \{b\}^{\{b\}}\,\mathrm{m}$ and $r_1 = \{b\}^{\{b\}}\,\mathrm{m}$. Further, the speed of light is precisely $299\,792\,458\,\mathrm{m\cdot s^{-1}}$.

Being realistic, you cannot really do power of those huge numbers, so let us rather notice that both the same, hence

$$I_2 = 1\,\mathrm{A}\cdot\frac{k+k}{k} = 2\,\mathrm{A}\,.$$

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Problem X.1 ... in the subway

There are two escalators, one is leading out of the subway station, the other one into it. They move in the opposite dirrections with velocities $0.5\,c$*, where* c *is speed of light in vacuum. Bob is in a hurry to get to the platform and he's running with velocity* $0.6\,c$ *with respect to the escalator. Bobek, on the other hand, is trying to get out of the subway with velocity* $0.4\,c$*, related to escalator. We are the observers standing on the platform. What is the velocity of their mutual movement according to our measurement?*

Dominika took a subway for the first time in her life.

The velocities are high enough so we have to think in relativistic terms (since we don't want to violate the laws of physics by moving faster than light). We use a following formula

$$\frac{u+v_i}{1+uv_i/c^2}\,.$$

u is the velocity of the escalator while v_i is velocity of Bob (Bobek) with respect to the escalator. We obtain two velocities

$$v_{\mathrm{Bob}} \doteq 0.85\,c\,,$$
$$v_{\mathrm{Bobek}} \doteq 0.75\,c\,.$$

Adding these two will give us velocity we are looking for

$$v = |v_{\mathrm{Bob}} + v_{\mathrm{Bobek}}| = \frac{u+v_1}{1+uv_1/c^2} + \frac{u+v_2}{1+uv_2/c^2} \doteq 1.596\,c\,.$$

Rounding this number we get $1.60\,c$.

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Problem X.2 ... crash

A proton with invariant mass of $938\,272.0\,\mathrm{keV}$ *and kinetic enegry of* $1\,\mathrm{MeV}$ *hits a nucleus of the isotope* ${}^{7}_{3}\mathrm{Li}$ *with mass of* $7.016003\,m_u$ *and induces a decay to two non-excited* α *particles with invariant mass of* $3.727379\,\mathrm{GeV}\cdot c^{-2}$*. What will be the total kinetic energy in* MeV *of these two particles? Consider* $m_u = 931.2720\,\mathrm{MeV}\cdot c^{-2}$ *and* $c = 299\,792\,458\,\mathrm{m{\cdot}s^{-1}}$*.*

Even f(Aleš) used to play marbles.

We use the energy conservation law which states that

$$T = T_{\mathrm{p}} + (m_{\mathrm{Li}} + m_{\mathrm{p}} - 2m_\alpha)\,c^2\,,$$

where T is the wanted kinetic energy, T_{p} is the kinetic energy of the proton, m_{Li} is the mass of the lithium nucleus, m_{p} is the proton mass and m_α is the mass of the α particle.

We convert everything to electronvolts and after evaluating the equation we get $T \doteq 18\,\mathrm{MeV}$.

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Problem X.3 ... blip

A particle has a mass increased sevenfold from its invariant mass due to its movement. You can follow the particle on a path $l = 1\,\mathrm{m}$ long with a measuring device. How fast must the device be to register the particle, i. e., what is the shortest time interval it has to distinguish in order to register the particle? The speed of light is $c = 299\,792\,458\,\mathrm{m{\cdot}s^{-1}}$. State the result in ns. *f(Aleš) writing when healthy, writing when sick.*

The energy of the particle is

$$E = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} = n m_0 c^2 \,.$$

From that follows that

$$\frac{1}{1 - \frac{v^2}{c^2}} = n^2 \,,$$

so that

$$v = \sqrt{c^2 \left(1 - \frac{1}{n^2}\right)} = \frac{c}{n}\sqrt{n^2 - 1} \,.$$

The particle flies through the distance l in time τ given by

$$\tau = \frac{l}{v}$$

$$\tau = \frac{l}{\frac{c}{n}\sqrt{n^2 - 1}} \,.$$

Evaluation gives $v \doteq 3.37021 \cdot 10^{-9}\,\mathrm{s}$ which is $v \doteq 3.4\,\mathrm{ns}$.

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Problem X.4 ... cracking right now

An unknown sample of rock contains $2\,\%$ of thorium, which contains $0.05\,\%$ of the ${}^{232}_{90}\mathrm{Th}$ radionuclide. The weight of the sample is $100\,\mathrm{g}$. What is the activity of this sample if the half-life of thorium is $1.4 \cdot 10^{10}$ years? Assume that nothing else decays except the mentioned thorium. *f(Aleš) recalled the loading of equipment for a camp.*

Let $a = 0.2\,\%$, $b = 0.05\,\%$ and $m = 0.1\,\mathrm{kg}$. The atomic mass of thorium is $A_\mathrm{r} \doteq 232$. The activity is given by

$$A(t) = \lambda N(t) \,,$$

where λ is the decay constant defined as $\lambda = \ln 2/T$ and N is the number of decaying nuclei. These are the nuclei of the radioactive thorium ${}^{232}_{90}\mathrm{Th}$, whose number can be find from the ratio of the weight of the radionuclide in the sample to the mass of one atom of ${}^{232}_{90}\mathrm{Th}$.

$$N = \frac{abm}{A_\mathrm{r} m_u} \,,$$

where A_r is the atomic mass of thorium and m_u is the atomic mass unit, explicitly

$$N = \frac{0.020.05 \cdot 10^{-2} \cdot 0.1}{232 \cdot 1.66 \cdot 10^{-27}} \doteq 2.60 \cdot 10^{18} \,.$$

If we consider that a year has $3.16 \cdot 10^7$ s, we get

$$A = \frac{\ln 2}{1.4 \cdot 10^{10} \cdot 3.16 \cdot 10^7\,\mathrm{s}} \cdot \frac{0.02 \cdot 5 \cdot 10^{-4} \cdot 0.1}{232 \cdot 1.66 \cdot 10^{-27}} \doteq 4.07\,\mathrm{Bq}$$

So the activity is about $4.1\,\mathrm{Bq}$.

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FYKOS is organized by students of Faculty of Mathematics and Physics of Charles University. It's part of Public Relations Office activities and is supported by Institute of Theoretical Physics, MFF UK, his employees and The Union of Czech Mathematicians and Physicists.